Fall18_HW06 — Recursive sequences, fixed points and limits

The handwritten assignment is **due** in recitation on **Thursday, September 20**.

When submitting a written homework you will be required to follow these guidelines:

- all pages for each assignment must be stapled together with a single staple in the top left corner;
- if you submit your homework on pages from a notebook, then you must remove the frilly edges;
- you must either (a) have all problems in numerical order or (b) start every problem on the left side of the page so that the TA can easily find the problems that s/he chooses to grade.

You will be penalized one point for each unsatisfied item listed above on every handwritten assignment.

1. (Problem # 85, p. 79)
Consider the sequence recursively defined by the relation

$$a_{n+1} = 3a_n - 2 \qquad a_0 = 1.$$

Compute a_n for $n = 1, 2, \ldots, 5$.

2. (Problem # 89, p. 79)
Consider the sequence recursively defined by the relation

$$a_{n+1} = \frac{a_n}{1 + a_n} \qquad a_0 = 1.$$

Compute a_n for $n = 1, 2, \ldots, 5$.

3. (Problem # 93, p. 79)
Consider the sequence recursively defined by the relation

$$a_{n+1} = \frac{1}{2}a_n + 2.$$

Find all fixed points of $\{a_n\}$.

4. (Problem # 97, p. 79)
Consider the sequence recursively defined by the relation

$$a_{n+1} = \frac{4}{a_n}.$$

Find all fixed points of $\{a_n\}$.

5. (Problem # 101, p. 79)
Consider the sequence recursively defined by the relation

$$a_{n+1} = \sqrt{5a_n}.$$

Find all fixed points of $\{a_n\}$.

6. (Problem # 103, p. 79)
Consider the sequence recursively defined by the relation

$$a_{n+1} = \frac{1}{2}(a_n + 5) \qquad a_0 = 1$$

and assume that $\lim_{n\to\infty} a_n$ exists.
Find all fixed points of $\{a_n\}$, and use a table or other reasoning to guess which fixed point is the limiting value for the given initial condition.

7. (Problem # 107, p. 79)
Consider the sequence recursively defined by the relation

$$a_{n+1} = 2a_n(1 - a_n) \qquad a_0 = 0.1$$

and assume that $\lim_{n\to\infty} a_n$ exists.
Find all fixed points of $\{a_n\}$, and use a table or other reasoning to guess which fixed point is the limiting value for the given initial condition.

8. (Problem # 108, p. 79)
Consider the sequence recursively defined by the relation

$$a_{n+1} = 2a_n(1 - a_n) \qquad a_0 = 0$$

and assume that $\lim_{n\to\infty} a_n$ exists.
Find all fixed points of $\{a_n\}$, and use a table or other reasoning to guess which fixed point is the limiting value for the given initial condition.

9. (Problem # 109, p. 79)
Consider the sequence recursively defined by the relation

$$a_{n+1} = \frac{1}{2}\left(a_n + \frac{4}{a_n}\right) \qquad a_0 = 1$$

and assume that $\lim_{n\to\infty} a_n$ exists.
Find all fixed points of $\{a_n\}$, and use a table or other reasoning to guess which fixed point is the limiting value for the given initial condition.

10. (Problem # 110, p. 79)
Consider the sequence recursively defined by the relation

$$a_{n+1} = \frac{1}{2}\left(a_n + \frac{9}{a_n}\right) \qquad a_0 = -1$$

and assume that $\lim_{n\to\infty} a_n$ exists.
Find all fixed points of $\{a_n\}$, and use a table or other reasoning to guess which fixed point is the limiting value for the given initial condition.